Shortcutting the diameter of point sets and convex polygons using a highway

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We investigate the problem of optimally placing a linear transportation facility, referred to as a *highway*, in the plane. A highway is a straight line segment, and one can traverse it at a high speed $v > 1$, but can only travel at a unit speed between any two points that are not on the highway. We consider two highway models: a *turnpike*, accessible only at its endpoints, and a *freeway*, accessible at any point on it. Also, we consider two domains: a point set and a convex polygon. See Figure 1.

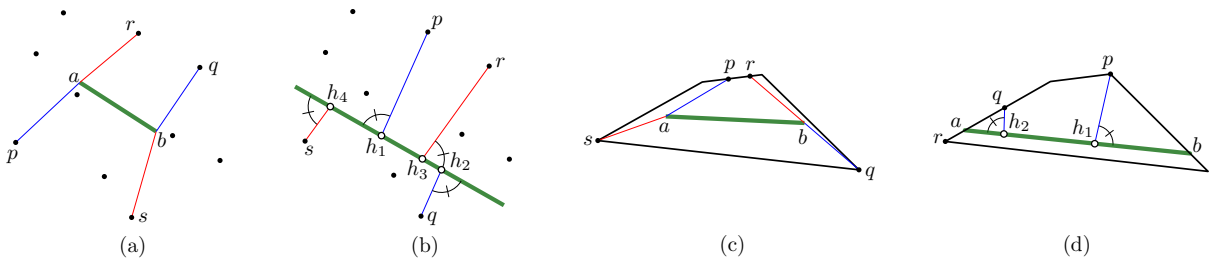

Figure 1: (a) An optimal turnpike in point set with the diametral pairs (p, q) and (r, s): diametral paths are pq and rs. (b) An optimal freeway in point set with the diametral pairs (p, q) and (r, s): diametral paths are ph_1h_2q and rh_3h_4s. (c) An optimal turnpike in convex polygon with the diametral pairs (p, q) and (r, s): diametral paths are $pabq$ (also pq) and $rbas$. (d) An optimal freeway in convex polygon with the diametral pairs (p, q) and (p, r): diametral paths are ph_1h_2q (also pq) and ph_1ar.

Our contribution. For a set of n points in the plane, we give randomized algorithms with $O(n \log n)$ expected time for placing an optimal turnpike under L_p metric for any fixed $p \in [1, \infty]$ and for placing an optimal freeway under L_2 metric. For a convex polygon with n vertices in the plane, we give randomized algorithms for placing optimal turnpikes and freeways lying entirely within the polygon under L_p metric, running in expected $O(n)$ time for any fixed $p \in \{1, \infty\}$ and in expected $O(n \log n)$ time for any fixed $p \in (1, \infty)$.

Our approach is based on *implicit quasiconvex programming*. We show that the travel time functions are quasiconvex with respect to highway parameters under certain constraints. By combining Eppstein's quasiconvex optimization framework with Chan's implicit LP-type method, we obtain near-linear (or linear) randomized expected time algorithms for all variants.

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